PLOS ONE

Footprints to Singularity: A global population model explains late 20th century slow-down and predicts peak within ten years.

--Manuscript Draft--

Manuscript Number:	PONE-D-21-03679
Article Type:	Research Article
Full Title:	Footprints to Singularity: A global population model explains late 20th century slow-down and predicts peak within ten years.
Short Title:	Systems dynamics models for global human population
Corresponding Author:	Christopher Bystroff Rensselaer Polytechnic Institute Troy, NY UNITED STATES
Keywords:	population; system dynamics; world modeling
Abstract:	Projections of future global human population are traditionally made using birth/death trend extrapolations, but these methods ignore limits. Expressing humanity as a K-selected species whose numbers are limited by the global carrying capacity produces a different outlook. Population data for the second millennium up to the year 1970 was fit to a hyper-exponential growth equation, where the rate constant for growth itself grows exponentially due to growth of life-saving technology. The discrepancies between the projected growth and the actual population data since 1970 are accounted for by a decrease in the global carrying capacity due to ecosystem degradation. A system dynamics model that best fits recent population numbers suggests that the global biocapacity may already have been reduced to one-half of its historical value and global carrying capacity may be at its 1965 level and falling. Simulations suggest that population may soon peak or may have already peaked. Population projections depend strongly on the unknown fragility or robustness of the Earth's essential ecosystem services that affect agricultural production. Numbers for the 2020 global census were not available for this study.
Order of Authors:	Christopher Bystroff
Additional Information:	
Question	**Response**
Financial Disclosure Enter a financial disclosure statement that describes the sources of funding for the work included in this submission. Review the submission guidelines for detailed requirements. View published research articles from *PLOS ONE* for specific examples. This statement is required for submission and **will appear in the published article** if the submission is accepted. Please make sure it is accurate.	C.B. was sopprted by an award from a private foundation that wishes to remain anonymous. The funders had no role in study design, data collection and analysis, decision to publish, or preparation of the manuscript.

Unfunded studies Enter: *The author(s) received no specific funding for this work.* **Funded studies** Enter a statement with the following details: • Initials of the authors who received each award • Grant numbers awarded to each author • The full name of each funder • URL of each funder website • Did the sponsors or funders play any role in the study design, data collection and analysis, decision to publish, or preparation of the manuscript? • **NO** - Include this sentence at the end of your statement: *The funders had no role in study design, data collection and analysis, decision to publish, or preparation of the manuscript.* • **YES** - Specify the role(s) played. * typeset	
Competing Interests Use the instructions below to enter a competing interest statement for this submission. On behalf of all authors, disclose any competing interests that could be perceived to bias this work—acknowledging all financial support and any other relevant financial or non-financial competing interests. This statement is required for submission and **will appear in the published article** if the submission is accepted. Please make sure it is accurate and that any funding sources listed in your Funding Information later in the submission form are also declared in your Financial Disclosure statement. View published research articles from *PLOS ONE* for specific examples.	The author has declared that no competing interests exist.

NO authors have competing interests Enter: *The authors have declared that no competing interests exist.* **Authors with competing interests** Enter competing interest details beginning with this statement: *I have read the journal's policy and the authors of this manuscript have the following competing interests: [insert competing interests here]* * typeset	
Ethics Statement Enter an ethics statement for this submission. This statement is required if the study involved: • Human participants • Human specimens or tissue • Vertebrate animals or cephalopods • Vertebrate embryos or tissues • Field research Write "N/A" if the submission does not require an ethics statement. General guidance is provided below. Consult the submission guidelines for detailed instructions. **Make sure that all information entered here is included in the Methods section of the manuscript.**	N/A

Format for specific study types **Human Subject Research (involving human participants and/or tissue)** • Give the name of the institutional review board or ethics committee that approved the study • Include the approval number and/or a statement indicating approval of this research • Indicate the form of consent obtained (written/oral) or the reason that consent was not obtained (e.g. the data were analyzed anonymously) **Animal Research (involving vertebrate animals, embryos or tissues)** • Provide the name of the Institutional Animal Care and Use Committee (IACUC) or other relevant ethics board that reviewed the study protocol, and indicate whether they approved this research or granted a formal waiver of ethical approval • Include an approval number if one was obtained • If the study involved *non-human primates*, add *additional details* about animal welfare and steps taken to ameliorate suffering • If anesthesia, euthanasia, or any kind of animal sacrifice is part of the study, include briefly which substances and/or methods were applied **Field Research** Include the following details if this study involves the collection of plant, animal, or other materials from a natural setting: • Field permit number • Name of the institution or relevant body that granted permission	
Data Availability Authors are required to make all data underlying the findings described fully available, without restriction, and from the time of publication. PLOS allows rare exceptions to address legal and ethical concerns. See the PLOS Data Policy and FAQ for detailed information.	Yes - all data are fully available without restriction

A Data Availability Statement describing where the data can be found is required at submission. Your answers to this question constitute the Data Availability Statement and **will be published in the article**, if accepted. **Important:** Stating 'data available on request from the author' is not sufficient. If your data are only available upon request, select 'No' for the first question and explain your exceptional situation in the text box. Do the authors confirm that all data underlying the findings described in their manuscript are fully available without restriction?	
Describe where the data may be found in full sentences. If you are copying our sample text, replace any instances of XXX with the appropriate details. • If the data are **held or will be held in a public repository**, include URLs, accession numbers or DOIs. If this information will only be available after acceptance, indicate this by ticking the box below. For example: *All XXX files are available from the XXX database (accession number(s) XXX, XXX.).* • If the data are all contained **within the manuscript and/or Supporting Information files**, enter the following: *All relevant data are within the manuscript and its Supporting Information files.* • If neither of these applies but you are able to provide **details of access elsewhere**, with or without limitations, please do so. For example: *Data cannot be shared publicly because of [XXX]. Data are available from the XXX Institutional Data Access / Ethics Committee (contact via XXX) for researchers who meet the criteria for access to confidential data.* *The data underlying the results presented in the study are available from (include the name of the third party*	All World4 model data are included in Table 1 of the manuscript. Population data is available from Worldometer (www.Worldometers.info). Within that source, from 1950 to current year, data is from United Nations, Department of Economic and Social Affairs, Population Division. World Population Prospects: The 2019 Revision. (Medium-fertility variant). Access to the latest World4 model through insightmaker.com is available by searching for "World4.5". Hyperfit.f90 is available upon request from the author.

and contact information or URL). • This text is appropriate if the data are owned by a third party and authors do not have permission to share the data. * typeset	
Additional data availability information:	

title:

Footprints to Singularity: A global population model explains late 20th century slow-down and predicts peak within ten years.

short title:

Systems dynamics models for global human population

Christopher Bystroff

Dept of Biological Sciences, Dept of Computer Science, Rensselaer Polytechnic Institute, Troy NY 12180

Abstract

Projections of future global human population are traditionally made using birth/death trend extrapolations, but these methods ignore limits. Expressing humanity as a K-selected species whose numbers are limited by the global carrying capacity produces a different outlook. Population data for the second millennium up to the year 1970 was fit to a hyper-exponential growth equation, where the rate constant for growth itself grows exponentially due to growth of life-saving technology. The discrepancies between the projected growth and the actual population data since 1970 are accounted for by a decrease in the global carrying capacity due to ecosystem degradation. A system dynamics model that best fits recent population numbers suggests that the global biocapacity may already have been reduced to one-half of its historical value and global carrying capacity may be at its 1965 level and falling. Simulations suggest that population may soon peak or may have already peaked. Population projections depend strongly on the unknown fragility or robustness of the Earth's essential ecosystem services that affect agricultural production. Numbers for the 2020 global census were not available for this study.

Introduction

Global human population has grown alarmingly in the 20th century, leading to speculation about the maximum number we will reach and when. Estimates of the human carrying capacity vary widely[45], as widely as the fields of study that address population: economics, demographics, history, system dynamics, ecology, sociology, archeology, and bioinformatics to name a few. Some see doom and gloom in our collective future[11,43], and rational discourse is inhibited by a broad and multifaceted taboo on discussion of population in general[44]. So global population dynamics remains understudied and poorly understood. The field lacks hard models and objective scrutiny, settling for informal, subjective and descriptive models that are poor predicters of the true numbers that we would like to know.

what does hard mean here?

This paper asserts two points. (1) Global human population growth does not fit the exponential growth equation that governs all living things under static growth conditions. Instead it fits a hyper-exponential function in which the growth rate itself grows exponentially[3]. The implication is that there is a quantity that governs the rate of growth, and that it is itself a growing quantity. *Technology*, broadly defined, is such a quantity, since it drives increases in human life expectancy[4]. Knowledge of technology grows exponentially because established knowledge enables and accelerates the emergence of new knowledge. (2) Obviously, the world is finite and population growth cannot go on forever, but within that truth it is uncertain whether population will top-out at the carrying capacity, exceed it, or crash to a lower level. The outcome depends on the nature of the carrying capacity. In this work, carrying capacity is defined as a *dynamic number of humans that cannot be exceeded,* consistent with some prior use[7], but other prior uses treat carrying capacity as a static number or a number that can be exceeded. A dynamic hard limit is consistent with the treatment of humans as a K-selected species with a food-supply-limited population[2]. The food supply is in turn impacted by environmental degradation, including climate change[8]. These two concepts, hyperexponential technology-driven growth and dynamic carrying capacity-limited population, are encoded in a new, predictive system dynamics model.

It cannot continue to grow exponetially!

Past predictions of the future of human population range from qualitative to quantitative. Demographic transition theory[9] asserts that technological advances decrease the death rate first, then decrease the birth rate, leading ultimately to stability. United Nations-sponsored extrapolations of trends in birth and death rates, sometimes including a stochastic treatment of migration, predict a population peak around 2050 at around 9 billion followed by a slow decline[1]. An older, surprisingly simple mathematical model fits population to a hyperbola[10]. Although presented in a tongue-in-cheek manner, the hyperbolic "doomsday" model nonetheless correctly predicted the world population within 8% for another 40 years after it was published in 1960. However, these models contain no explicit global limits.

The ecological footprint literature sets a numerical limit to the sustainable human impact on the planet[11]. If the "footprint" is not maintained within sustainable limits, then an increase in the mortality rate ensues. However, this negative feedback loop is only informally described. A key concept in moving from an informal to a formal (*i.e.* numerical and predictive) model is the notion that nature renders essential services to humanity in proportion to nature. Herein, the *ecosphere* is defined as the portion of the global biocapacity that is not appropriated by humans and ~~which~~ that renders the essential ecosystem services. The *humansphere* is defined as the portion of the biocapacity that has been taken from the *ecosphere* and does not yield these services. These two terms come from the footprint literature[12], but here they are conceived with a few subtle differences. The *ecosphere,* but not the *humansphere,* absorbs carbon dioxide from the atmosphere to the extent that it contains growing plants. Absorption of carbon dioxide by the *humansphere* does not count because carbon dioxide absorbed into food by agriculture is re-emitted by decomposition or after eating by respiration. Likewise, the *ecosphere*, but not the *humansphere*, regenerates fresh water, regenerates soil fertility, pollinates flowering plants, and stabilizes the climate roughly in proportion to its fraction of the Earth's biocapacity. It also provides numerous support services "behind the scenes", such as maintenance of the food web that supports fish stocks and maintenance of the habitats of pollinating insects. To the extent that these services are lost, land from the *ecosphere* passes to the *humansphere*.

Technology and its effect on population has been written about extensively. Some view technology as an outgrowth of increased population ("More people means more Isaac Newtons")[40,41] and others see it as a driver of growth[42]. Among those who see technology as a causal agent, some see it as an intrinsic property of all life, growing as if it were a living thing[43]. The so-called "singularity" is viewed as a point where technological advancement escapes human control[46]. To formalize the concept and make it predictive and numerical, we must define technology and attach it to something that we can measure. Herein, technology is defined as the capacity to decrease the death rate and to increase the carrying capacity. As such, technology is attached to the population growth rate.

System dynamics (SD) models for world population dating back to the 1970's included explicit limits to growth using ecological and economic feedback loops, providing a means to reproduce technology-driven hyper-exponential growth and to forecast a downturn[13,14]. A 2004 "world" SD model called World3 predicted our population future under various policy scenarios[15]. Its "standard run" predicted a peak population between 9 and 14 billion happening between years 2075 and 2085, followed by a decline of 20 to 40% over the subsequent 50 years, depending on estimates for global arable land. Several other world models were developed in the wake of World3, each of which endeavored to make the system more complex, not simpler. Authors of some follow-up studies chose to subdivide humanity[16]; others introduced complex resource management systems[17]. The perceived complexity of SD models may have impeded their widespread adoption.

what does this mean?

Modeling humanity as a K-selected species with technological acceleration of growth and a proportional degradation of the food production system leads inevitably to a boom/bust outcome. Consider that, first, humanity cannot exceed the food supply; second, that the food supply depends on ecosystem services that are essential for their production; and third, that those natural systems are being destroyed by human growth. Positive followed by negative feedback all but assures an overshoot followed by a crash in population, not a high plateau as some have predicted, although this is still possible.

This paper presents the arguments for this pessimistic projection. A formal model is used to explore the ~~dark~~ space of population collapse in detail, and future projections are fine-tuned by optimizing the model parameters against past population numbers and other data. Concepts derived from or inspired by the footprint literature[2], the theorized technological "singularity"[43], and the biological view of humanity as a K-selected species[2], have led to the system dynamics model presented here (Figure 1) . This is a "mind size"[18] SD world model, offered with the hope of making a complex system simpler, easier to understand

and easier to teach, and to better understand the Earth's limits to growth and humanity's likely future trajectory.

Figure 1. (a) World4, a system dynamics model that reproduces world population numbers up to 2010 and projects forward. Stocks (rectangles) and flows (solid arrows) form two interacting closed systems, one for Technology and one for Environment. Input variables (ovals) are colored and grouped by function. Output variables (white) are the global carrying capacity (*CC*) and *population*. Dashed lines indicate variable dependencies. (b) World4 simulations superposed on 20th century population numbers (thick cyan line) and UN population projections[1] (dashed blue line is the median projection and light blue are 95% confidence region). The program *hyperfit* carried out 1 million World4 simulations using randomly selected parameters from ranges listed in Table 1. Shown are the 184 trajectories that deviate from 1970-2010 population data by less than *rms* 0.5e8. Simulations are colored by their $\boldsymbol{E_0}$ value (total ecosystem size in *gha*, see inset). Counterintuitively, a low $\boldsymbol{E_0}$ means a higher population is sustainable.

~~Results~~ what follows is not results. It is mainly methodology.

The simple model, called World4, consists of four stocks and four flows, forming two interacting binary subsystems. There are two stocks quantifying states of the global environment (*humansphere* versus *ecosphere*) and two stocks quantifying states of technological development (*knowledge* versus *ignorance*). Both are closed systems, which means there are no outside sources or sinks. The only source of domesticated land (*humansphere*) is wild land (*ecosphere*), and life-saving technologies (*knowledge*) can only save more lives to the extent that there is a non-zero death rate (*ignorance*). In this model, *knowledge* draws down the death rate, while *ignorance* is the death rate itself. *Knowledge* does double duty by also increasing the carrying capacity. Other forms of knowledge and technology are ignored. We'll get back to technology, but first let's discuss humanity.

Environment

Humanity is represented by its ecological footprint within the Environment subsystem. The footprint is the amount of Earth's biocapacity appropriated for human use[12]. The Earth has a maximum total biocapacity ($\boldsymbol{E_0}$) estimated to be around 1.12e10 global hectares (gha) ~~which~~ that is shared between wild and domesticated land and sea. *Humansphere* is equal to population times the average consumption per capita, times a term derived from the state of technology. In terms of the popular "I=PAT"[47] for ecological impact, *humansphere* is "I", *population* is "P", and the carrying capacity equation (*CC*) expresses the "A" (affluence) and "T" (technology) terms. In this model, *population* is viewed as a consequence rather than a cause of consumption. Therefore the expression becomes P=I/(AT), or

$$\textit{population} = \textit{humansphere} \times \textit{CC}, \qquad (1)$$

where *CC* is the reciprocal encoding of the informal concepts *A* and *T*. *Humansphere* grows intrinsically in the model, via the flow

$$\textit{domestication} = (\boldsymbol{I_0} + \ln(2)/\boldsymbol{\tau})\,(1\text{-}\exp(\boldsymbol{u}\ \textit{ecosphere}/\boldsymbol{E_0})) \times \textit{humansphere} \qquad (2)$$

because human need for more land grows with, and is proportional to, the population. The rate constant ($\boldsymbol{I_0} + \ln(2)/\boldsymbol{\tau}$) (1-exp($\boldsymbol{u}$ *ecosphere*/$\boldsymbol{E_0}$) approaches zero as *ecosphere* approaches zero because *ecosphere*

I assume u is constant?

cannot be negative. $\boldsymbol{u}$ is a negative number that models the strength of this feedback. $\boldsymbol{I_0} + \ln(2)/\boldsymbol{\tau}$ is the birth rate, which is the replacement of deaths ($\boldsymbol{I_0}$) plus the net growth, where the latter is reciprocally related to the intrinsic doubling time $\boldsymbol{\tau}$, in years. The approach of basing population on carrying capacity is contrary to most population models, including World3[14], that express population in terms of births and deaths. Hopfenberg[2] and others have argued that population growth should be viewed as food-supply driven rather than the result of births and deaths. This view has been met with skepticism[48] but it is consistent with the treatment of humans as a K-selected species. To treat humans differently from all other K-selected species would be a form of "human exceptionalism" [49], which is not scientific. Moreover, *ecosphere* cannot be expressed in terms of a population, so *gha* units make sense for these stocks.

Technology

In this model, technology is a driver of growth through the suppression of mortality and is a factor in amplifying the human carrying capacity. In a sense, technology (or knowledge of technology) is an independent living entity since it can exist in written form outside of humanity itself and since it has its own catalytic effect on the development of new technology. Thus it is not wrong to treat technology as a living thing that can grow exponentially. The expression for intrinsic flow into *knowledge* is

$$learning = \kappa \times knowledge \tag{3}$$

where κ is a constant. Also, like a living thing, technology can "die" by obsolescence. Even though the knowledge may still remain, its usefulness towards human survival can whither to zero (buggy whip technology, for instance). Thus in the model we lump together unlearned or unknown technology with obsolescent or ineffective technology under the term *ignorance*, a quantity that is rate constant for mortality. More specifically, since humanity is being expressed in terms of its ecological footprint in *gha* units, and because death represents the return of human appropriated biocapacity to the *ecosphere,* therefore *ignorance* is the rate constant for that flow, called *rewilding.*

$$rewilding = ignorance \times humansphere \tag{4}$$

Feedback loops

The two subsystems, Environment and Technology, interact to form a feedback loop. It is postulated that the loss of the undomesticated environment will cause technological challenges to human survival due to the loss of ecosystem services that are required for food production. To model this, *ecosphere* feeds back to *obsolescence* as follows.

$$obsolescence = 0.5 \exp(\boldsymbol{\nu}\ ecosphere/\boldsymbol{E_0}) \times knowledge \tag{5}$$

where $\boldsymbol{\nu}$ is a negative number. As the wild environment disappears (depleting fresh water aquifers and fossil fuels, for instance), old technologies lose their usefulness (center-pivot groundwater irrigation systems, oil-fired electric generators), and humanity must develop new technologies to survive (ocean desalination, photovoltaic panels). Thus it makes sense that *ecosphere* depletion leads to *obsolescence*. This completes a negative feedback loop (Figure 2a). To play out a scenario on this loop, we can imagine that *ignorance* goes to zero, therefore *rewilding* decreases. This leads to a decrease in *ecosphere* by *domestication.* Depletion of *ecosphere* causes an increase in *ignorance*, therefore an increase in *rewilding*, reversing the effect. In the context of the exponentially depleting quantities *ignorance* and *ecosphere*, the system reaches a switching point and thereafter equilibrates (Figure 2b). Interestingly, this

This needs further amplification and a reference

is the same feedback cycle that generates Lotke-Volterra oscillation (inset in Figure 2b) but under other circumstances. ~~Because~~ As both *ecosphere* and *ignorance* are decreasing rapidly, the oscillation is severly damped to the point of being singular.

Parameter

Fitting

Be clear what the parameters are and their specific ranges

Figure 2. **Feedback**. (a) *Ignorance* feeds back in a positive way to *rewilding*. *Rewilding* increases *ecosphere*. *Ecosphere* feeds back negatively to *obsolescence*. *Obsolescence* increases *ignorance*. (b) Exponential decrease of both *ecosphere* (pE) by *domestication* and *ignorance* (pI) by learning, results in a switch, first in *ignorance* then in *ecosphere*. Inset: undamped Lotke-Volterra oscillation.

Parameter settings for the model were determined by non-linear least squares fitting against historical population data. The intent of the project was to explain the hyper-exponential growth that was observed in the 19th and 20th centuries, and to predict the outcome of the ever-widening downward discrepancy between the expected hyper-exponential growth and the actual population trajectory since 1970 (Figure 3d). Fitting data before 1970 to a hyper-exponential model asks the question “How did humanity grow in the absence of limits?” Whereas, fitting the late 20th century discrepancy asks the question "How do planetary limitations slow the growth of humanity?" The results of the fitting are numbers defining the empirical upper limit for population and an expression for how the degradation of our environmental support system feeds back on that upper limit.

You should have defined this before

There should be a sensitivity analysis. I see it is done later

Figure 3. Least-squares fits to years 1970 - 2010 are non-linearly correlated in the space of the four variables ($\boldsymbol{E_0}$, $\boldsymbol{a}$, $\boldsymbol{u}$ and $\boldsymbol{v}$) that effect only recent population data, as shown using *hyperfit*. For example, as seen in (a), the best-fit setting for $\boldsymbol{v}$ (ecosystem-dependent obsolescence of technology) goes down as we increase the setting for $\boldsymbol{a}$ (ecosystem fragility). Each image is a projection of minimum values of residuals from the 4D space to 2D spaces (a) $\boldsymbol{a}$, $\boldsymbol{v}$, (b) $\boldsymbol{a}$, $\boldsymbol{E_0}$, (c) $\boldsymbol{E_0}$, $\boldsymbol{v}$. (d) A plot of five trajectories using optimal and suboptimal values, demonstrating the effect of choice of $\boldsymbol{E_0}$ (total ecosystem size) on growth rate (1960-2000) and on the position of the population peak, ignoring other parameters. A hypothesized infinite ecosystem, $\boldsymbol{E_0}=\infty$ (black), leads to massive overestimate of growth rate. $\boldsymbol{E_0}$=0.800e10 (green) or $\boldsymbol{E_0}$=0.750e10 (orange) overestimates growth rate and predicts a later peak. $\boldsymbol{E_0}$=0.695e10 (cyan) is optimal and predicts a 2020 peak. $\boldsymbol{E_0}$=0.600e10 (magenta) underestimates growth and predicts that we are past the peak. Thick blue line is population data up to 2010. 2020 population numbers are not available.

After fitting, the output of this model matches all historical population data from years 1500 to 2010 within ±10% with the exception of the 1950 census, which is overestimated by 14%. It should be noted that 1950 census number was revised 17 times from 1951 to 1996, mostly upward[19]. Population growth has been sub-exponential over the last 50 years, suggesting that humanity is passing through an inflection point of a curve that is the product of two steep trends, one upward, the other downward. The upward curve is the combined exponential expansion of humanity and the intrinsically exponential increase in technological innovation, and the downward curve is the accelerating depletion of non-renewable resources and the loss of food security. The model predicts that the results of the 2020 census (not yet available) will be in the range 7.0 to 7.6 billion instead of the projected 7.8 billion[1]. The model predicts that the population may be peaking now and may likely decline to between 1 billion and 6 billion by 2100. The nearness of the peak is supported by accelerating increases in adult mortality and decreases in birth rates since 2016[6].

That is a huge difference. Discuss

The system dynamics simulation assumes fixed set of parameters and equations throughout the simulated time period. The model has no outside sources or sinks. There are no built-in switches and no settings are changed at any point in the simulation. Nonetheless, the simulation matches very closely to 2010 years of population history spanning periods of slow growth, rapid growth, and recent deceleration. The results of the study are a set of meaningful parameters that attach numbers to well-established but theoretical human/environment feedback mechanisms. Parametric solutions from the multi-variable least squares fit span only a narrow range in most variables, varying for the most part in the parameters that define the feedback between human population and the environment. Multiple solutions fit and explain past population data equally well but project very differently into the future (see examples in Figure 1b).

Variables were solved in three stages. First, intrinsic growth parameters ($\boldsymbol{H_0}$, $\boldsymbol{K_0}$, $\boldsymbol{\tau}$, and $\boldsymbol{\kappa}$) were fit to years 1 - 1970. Second, the three "footprint" variables that affect the balance between normal exponential growth and hyper-exponential growth (***b, c*** and ***d***) were fit to years 1900-2010. Third, the late 20th century discrepancy (1970 - 2010) was reconciled using the variables ($\boldsymbol{E_0}$, ***a, u, v***) that determine the total global biocapacity, the fragility of ecosystem services, and the strength of feedback from the environment on the carrying capacity for humans. One variable, the baseline mortality rate in Year 0 ($\boldsymbol{I_0}$) does not have an effect on population until after the present date, and therefore could not be fit. $\boldsymbol{I_0}$ may be arbitratilly set to 0.11 y^{-1} to reproduce previously estimated overshoot values (1.7 earths[50]). With this setting, the current population is 1.7 times the equilibrium value, which would be around 4 billion in 2100 using $\boldsymbol{I_0}$=0.11. Also, four variables ($\boldsymbol{w}$, $\boldsymbol{p}$, $\boldsymbol{s_y}$, $\boldsymbol{p_y}$) were created for the hypothetical implementation of environmental conservation policy in the 21st century.

Perhaps the most consequential result of this modeling is the prediction of a peak in population occurring within a narrow time range at or near the present time, followed by a steep decline. Exploring the ranges of the parameters $\boldsymbol{E_0}$, ***a, u,*** and ***v*** (see Figure 3a,b,c) that govern the negative feedback loop shown in Figure 2, we see a narrow range in the possible peak populations (7.0 to 8.0 billion) and peak population years (2016 to 2040) that are consistent with the population data. Peak populations and peak years outside of the range are not seen when the variables are well fit to data. The range of solutions presented in Figure 1b shows that possible 2060 populations vary from as low as 0.5 billion to as many as 6.5 billion. Longer term predictions are less confident.

This is not particularly showing confidence!

The behavior of the model with respect to parameters is complex and surprising. For example, if $\boldsymbol{E_0}$ is set to 7.50e9, it places the peak population year at 2038, but then the model fits poorly to the data, showing an upwardly curving trajectory through the 1980's and 90's (see Figure 3d), though we know that did not happen. Setting $\boldsymbol{E_0}$ to a low value of 5.5e9 places the peak at 2016 and shows a downward curving population trajectory from 1990 to 2010, which again we know did not happen. But if all other parameters are allowed to float and are fit to the data by exhaustive multidimensional search, then both high and low settings of $\boldsymbol{E_0}$ match very well to the 1970-2010 data (see Figure 1b). However, the results are reversed. The peak for low $\boldsymbol{E_0}$ is now at 2040, and the peak for high $\boldsymbol{E_0}$ is now at 2016. Surprisingly, in the context of the full complement of model parameters, the effect of $\boldsymbol{E_0}$ has become completely counter-intuitive. With a little effort we can rationalize this strange behavior, as follows. If $\boldsymbol{E_0}$ is high (8.0e9 gha) and yet population data is well fit, the parameter $\boldsymbol{a}$ adopts a high value ($\boldsymbol{a}$=0.48), which means the ecosystem is fragile and fails well before it is completely depleted. This leads to a steeper downturn in carrying capacity, which leads to the observed prediction, a counterintuitive earlier peak. On the other hand, if $\boldsymbol{E_0}$ is set to a low value ($\boldsymbol{E_0}$=4.2e9) and yet population data is well fit, then the parameter $\boldsymbol{a}$ adopts a low value ($\boldsymbol{a}$=0.10) and a later population peak is predicted. Low $\boldsymbol{a}$ is interpretted to mean that the ecosystem is robust and the ecosystem services upon which we all depend are not entirely provided by the wild *ecosphere* but may be partially provided by the *humansphere*. To be clear, these very different predicted futures are not scenarios that can be affected by policy change, rather they are the results of uncertainty and a lack of undertanding of the fragility or robustness of the global ecosystem with respect to human carrying capacity. Upcoming results from the 2020 census will greatly resolve this uncertainty.

The theoretical appearance of a population decline in the near future is a foregone conclusion of the design of the model itself. The model encodes the business-as-usual (BAU) assumption that humanity will not react to change and will continue to degrade the environment. It also assumes that the carrying capacity depends critically on resources that are not under human control nor are regenerated by human activity, and which will not come under human control within the timeframe of the simulation. These model design choices are not to be considered incontrovertible, but they are consistent with the dominant theory in human ecology and are intended to be free of human exceptionalism.

Discussion

Human population dynamics has origins in the environment and in human behavior. An understanding of the essence of our interactions with Nature and with each other can be gained by expressing human population in a simple dynamic model.   The approach taken here has been to define the components and equations of a systems model and to tune the parameters of these components to fit population data. Prior to the current model we explored a number of alternative models or equations and rejected them, either because they could not be tuned to fit the data, or because they did not make real world sense, or because they were too complex.

The World4 model is a BAU projection in the sense that human growth behavior and behavior towards the environment are treated as parameters that are solved from past population data, not parameters that seek an outcome. A range of parametric solutions has been found. The median projection (Figure 1)

This seems very unlikely

peaks at or around 2022, then falls steeply, leveling off at around 3 billion. The cause of the decline within the model is a decrease in the food supply caused in turn by degradation of the environment and the concomitant atenuation of essential ecosystem services. The model reflects the current thinking on climate change and its repercussions. Climate change leads to weather uncertainty, increased severe storms, draught and floods, and sea level rise affecting low-lying areas -- each a factor in decreasing agricultural output. Increased hunger in turn fuels conflict [20,21]. Conflict leads to further decreases in food production and to mass migration, as we have seen recently from the rapidly heating Sahel region of Africa[22,23]. In the BAU projection we see an increase in human mortality, followed by a decrease in carrying capacity. The recent worldwide spread of Covid-19 is an example of an emerging source of mortality for which human technology was not ready. Societal stressors such as hunger or a pandemic can drive violent behavior[24]. In the medan projection, following the peak, population drops quickly, accelerating to 100 million net lives lost per year through the years 2030 to 2040, which is faster than the fastest growth during the 20th century. In this model, we clearly see the cause of the rapid decline -- the exponential growth of the consumption of finite vital natural resources.

This is very subjective. For example arguably a move away from animal-based food production can be very significant.

We are beginning to see the effects of the decline of natural resources and ecosystem services. Fossil fuels, fresh water aquifers, and greenhouse gas sequestration by plants are all regarded in this model as part of the *ecosphere* since they are not generated by human activity. The decline of one or more natural resources is cited as a cause of, for example, the ongoing deadly conflicts in Syria starting since 2011, the conflicts and famine in Yemen beginning in 2015, and the economic collapse in Venezuela that began in 2014, to mention a few. Draught and desertification were blamed for conflicts and mass migration out of the Sahel region of Africa, where Lake Chad has all but disappeared. Conflicts and famine have produced millions of refugees. Innumerable lives have been lost crossing the Sahara or crossing the Mediterranean, or in primitive camps along the southern borders of Europe. The 2017 documentary film "Human Flow"[26] reveals the massive scale of the refugee issue. Meanwhile, the global north has responded to the aggregate changes of the last 50 years with a dramatic decrease in the birth rate. An increasingly technological workforce has meant women spend more time in school and marry later. Rising oil prices have steadily ramped up the cost of raising a child, leading to smaller families by choice. Total fertility rate (TFR) globally is projected to reach replacement level (2.11) this year, 2020.

Along with ecosystem decline, the model predicts changes in technology. In the projection, knowledge will be lost or made obsolescent during a population collapse. Much of our cultural technology is composed of laws, governance and economics. In recent economic history, consumerism has become engrained in our culture[27,28]. Stability and prosperity in the context of the current economic system relies on population growth, according to economists. A technology shift in economics is likely when population begins to decline, since growth-based economic systems and the associated body of knowledge will become obsolescent in the sense that they will not produce stability. In effect, economics will have to be re-learned. Obsolescence of growth-based economics may manifest itself in real-world breakdown in economic systems leading to decreased efficiency in manufacture and trade, in turn leading to a decrease in the effective food supply, which in turn will cause an increase in malnutrition and a decreased birth rate. Already, increased adult mortality and decreased birth rate are both current trends in global vital statistics[29]. In other areas, medical technology is partially responsible for a historic low death rate worldwide, but successful treatment of disease requires instruments and drugs that depend on a complex supply chain and high level engineering skills. In the event of an economic shift, supply chains will be disrupted unless a new system of economic motivation is quickly invented to replace the growth motive. In agriculture, technology to increase crop yields will become obsolescent as climate challenges, biodiversity losses (especially the loss of pollinating insects), and depletion of freshwater aquifers, combine with economic changes to reduce the efficiency of food production and distribution.

who could have foreseen the very rapid development of COVID19 vaccines!

But the future could easily be different. Humanity could adapt in many ways, good and bad. Modeling adaptation mechanisms opens a non-BAU modeling space that is too large to thoroughly explore. Taking inspiration from E. O. Wilson's book "Half Earth"[37], parameters for policies to preserve wild nature were implemented. Four new variables were added, ***w, y, py*** and ***sy***, as defined in Table 1, for the target amount of *ecosphere* to save, the level of policy enforcement, the phase-in period and the date on which the policy begins, respectively. These variables do not affect populations prior to and including 2010. Preserving wild land wild allows humans to thrive. The optimal result (coindidentally it is ***w***=0.5, half earth!) gives, as Wilson predicted, a stable and high human population (Figure 4). This makes sense mathematically, because the carrying capacity equation contains a term of the form $x(1-x)$, which has a maximum at $x = 0.5$ where x is the fraction of the Earth dedicated to the *ecosphere*. But it also makes sense ecologically, because maximum sustainable food production is a trade off between maximizing arable land and maximizing climate stability, the latter embodied in wild forests and arctic ice, and other buffers to change. A global climate awareness campaign might lead to such a balance.

Figure 4. Half-Earth scenarios. Population projections for conservation efforts with various % enforcement, length of phase-in period and the % of ecosphere to be preserved, as compared to a BAU scenario. Dotted line is the present year, 2020.

Table 1. Complete component list for World4 model in four parts. (a) Variables. (b) Flows. (c) Stocks. (d) Equations. Variables in bold italics were fit to data. Best value is one solution of many. Range shows values that can be fit to data with less than a specific residual depending on range of years fit. Fit years is the range used for fitting in *hyperfit*.

(a)

Var.	Best value	Range	Fit years	Physical meaning
a	0.426	0.35 to 0.48	1970-2010	Ecosystem fragility. Relates cc_E vs *ecosphere*. A higher/lower ***a*** means that ecosystem services are fragile/robust with respect to *ecosphere,* respectively
b	1.0 people/gha	0.7 to 1.7	1000-1970	Base level carrying capacity for *ecosphere*.
c	5.5 people/gha	4.5 to 7.0	1000-1970	Base level carrying capacity for *humansphere*.
d	-110	-150 to -90	n/a	Rule of diminishing returns. Relates *knowledge* to *CC*. A more negative value for ***d*** means *knowledge* raises *CC* more.
$\boldsymbol{E_0}$	7.05E+09 gha	4.3e9 to 8.1e9	1970-2010	Initial biocapacity of the *ecosphere*.
$\boldsymbol{H_0}$	1.5e8 gha	1.2e8 to 1.6e8	1-1970	Domesticated land in 0CE. Initial value of *humansphere*.
$\boldsymbol{I_0}$	0.05 y^{-1}	0.05 to 0.25	n/a	Base mortality. Multiplied by *humansphere* to get *rewilding*. Must be higher than maximum value of *knowledge*. Past population is insensitive to this variable but it affects future population.
$\boldsymbol{K_0}$	7.25e-11 y^{-1}	2.0e-11 to 2.0e-9	1000-1970	Technology in Year 0. Initial value of *knowledge* in 0CE.
p	n/a	0 to 1.0	n/a	Enforcement level of conservation policy. Higher ***p*** means stronger enforcement of policy.
py	n/a	0 to inf.	n/a	Policy phase-in time of conservation policy. Linear phase-in for enforcement of conservation policy *w*.
sy	n/a	1960 to inf.	n/a	Starting date of phase-in of conservation policy. When $p_E < w$, *domestication* is multiplied by $g = g(((y-sy)/py)p+(1-(y-\boldsymbol{sy})/py)(1-\exp(-10(w-p_E))) + \exp(-10(w-p_E))$, where y is the current year. Used only in the phase-in period *sy* through *sy*+*py*.
u	-8.6	-inf. to -6.5	1970-2010	Aggressiveness of growth.
v	-11.46	-inf. to -9.0	1970-2010	Aggressiveness of technological development.
w	n/a	0 to 0.5	n/a	Fraction of *ecosphere* to save using conservation policy. When $p_E < w$, *domestication* is multiplied by $p\,(1-\exp(-10(w-p_E))) + \exp(-10(w-p_E))$
$\boldsymbol{\kappa}$	9.6E-03 y^{-1}	6.5e-3 to 1.0e-2	1000-1970	Learning rate. Rate of the intrinsic growth of *knowledge*.
$\boldsymbol{\tau}$	852 y	700 to 1525	1-1970	Doubling time of *humansphere* in Year 0.

(b)

Flow	Source	Sink	Formula	Physical meaning
rewilding	*humansphere*	*ecosphere*	*ignorance*humansphere*	Deaths expressed as change in ecological footprint.
domestication	*ecosphere*	*humansphere*	*g*humansphere*	Births expressed as change in ecological footprint.
learning	*ignorance*	*knowledge*	$\boldsymbol{\kappa}$ **knowledge*	Intrinsic technology growth.
obsolescence	*knowledge*	*ignorance*	*r*knowledge*	Loss of technology.

(c)

Stock	Initial value	Physical meaning
humansphere	$\boldsymbol{H_0}$	Amount of total biocapacity appropriated for human use in Year 0, in gha.
ecosphere	$\boldsymbol{E_0}$	Amount of total biocapacity not appropriated for human use in Year 0, in gha.
knowledge	$\boldsymbol{K_0}$	Mortality eradicated by technology, in per year rate units y^{-1}.
ignorance	$\boldsymbol{I_0}$	Base mortality rate. Eradicated by technology. In per year rate units, y^{-1}.

(d)

Equation	Formula	Physical meaning
cc_E	$\boldsymbol{b}\, p_E^{(0.5/(1+pE-2a))}$	Carrying capacity contributed by the ecosphere.
cc_H	$\boldsymbol{c}\,(1 - \exp(\boldsymbol{d} * knowledge))\, \boldsymbol{cc_E}$	Carrying capacity contributed by the humansphere .
p_E	$ecosphere/\boldsymbol{E_0}$	The wild fraction of the environment.
g	$(\boldsymbol{I_0} + \ln(2)/\boldsymbol{\tau})(1-\exp(\boldsymbol{u}\, p_E))$	ecosphere-dependent net intrinsic growth rate of humansphere
r	$\exp(\boldsymbol{v}\, p_E)$	ecosphere-dependent depletion rate of knowledge
CC	$cc_E + cc_H$	Global carrying capacity in humans per gha.
population	*CC*humansphere*	Carrying capacity determines population number.

Any attempt to halt growth has to address the population. Cultural taboos currently prevent discussing, much less solving, this problem[44]. But in a what-if scenario, we can imagine ways that population growth can be halted or even reversed while preserving peace and prosperity. To build a mental picture of a society that has achieved balance with nature, imagine a people with a strong religious prohibition against growth, so engrained that no policing is required. A woman of child-bearing age in the Half-Earth world are permitted to have another child only if she is "blessed" by an elderly person, who, on his deathbed, bequeaths to her his one and only "blessing" -- the right to procreate. The one-to-one matching of deaths to births would guarantee population stability.

I still do not get a full view of the mathematical model

Conclusions

Hyperexponential population growth implies a dynamic system with two intrinsically growing quantities, human impact and technology. In a closed system, growth of one quantity implies depletion of another. *Knowledge* depletes *ignorance*, and the *humansphere* depletes the *ecosphere*. The quantities that are being depleted affect each other in a negative feedback loop leading to a sharp peak followed by a collapse of *humansphere* and *knowledge*. The timing of the collapse was determined by fitting the global limits to population in the context of hyperexponential growth. Population is predicted to peak between years 2017 and 2033 at a value of between 7.2e9 and 7.8e9 people. A much clearer picture will emerge when the 2020 census data is available.

Methods

Structure of the Model

The structure of the paper is very strange. Results before model description does not help in reading this paper.

It would be clearer if this is written as

$\log P = \log a + bt + ce^{dt}$

The hyper-exponential growth of human population from 1000 CE to the present is revealed by simple curve fitting to historical population data.

A possibility to get a better fit in (20) is

$\log P = \log a + bt + ce^{dt^{\alpha}}, \alpha \in [0,1]$

$$\boldsymbol{P_t} = 1.36e8 \exp(7.78e\text{-}4\, \boldsymbol{t} + 8.10e\text{-}9 \exp(9.74e\text{-}3\, \boldsymbol{t})) \quad (6)$$

So for $\alpha < 1$ the last term has slower exponential growth.

where t is the year since 0 CE and $\boldsymbol{P_t}$ is population at t. This formula fits all population numbers from 1700-1987 within 10%, and all numbers from the present back to 1000CE within 17%, except 2010. Note that this equation differs from an earlier published one (Eq 12 in [3]) with the addition of a log-linear term ($7.78e\text{-}4\, \boldsymbol{t}$). This term improves the fit to numbers in the Renaissance period. Eq. 6 may be interpreted as exponential growth in which the growth rate grows exponentially due to an exponential decline in the death rate. Other functional forms of the growth equation were considered and discarded, either because they did not fit the data or because they were not realistic. For example, normal exponential growth fits poorly to the data after 1500. On the other hand, a hyperbolic function fits all the data but lacks a physical rationale. Only the hyper-exponential equation achieves the steepness of human population growth in the 20th century without implying an unrealistic asymptote or arbitrary timepoints of discontinuity.

why that only?

But you say later model (6) only accurate for 1960. Many inconsistencies.

The hyper-exponential equation has two intrinsic growth rates. Therefore it implies two interacting subsystems, one containing the human population and the other containing a quantity that affects the rate of human growth. For this quantity we use the term "*knowledge*", meaning knowledge of technology. Technology generally improves life expectancy and efficiency in the use of natural resources and thus accelerates growth[4]. Knowledge of technology grows intrinsically, while humanity grows both intrinsically and extrinsically, depending on knowledge.

Drawing inspiration from the "ecological footprint" literature, the stock quantity that includes humanity is modeled as the total human-dominated portion of the global ecosystem, called the *humansphere* [12]. The remaining global biocapacity is the *ecosphere*, representing the portion of the global environment that is not under human domination. *Humansphere* and *ecosphere* are measured in global hectares (gha) and sum to a constant, which is the maximum total biocapacity, $\boldsymbol{E_0}$. Humans are assumed to be a K-selected species as opposed to r-selected[2]. The population of a K-selected species is determined by the carrying capacity, therefore multiplying *humansphere* by the carrying capacity per gha gives the population.

Growth of humanity is modeled as a flow of gha from *ecosphere* to *humansphere,* with value *domestication* = $\boldsymbol{g}$ x *humansphere*. This flow goes to zero as the *ecosphere*, the space into which *humansphere* must expand, goes to zero.

$$g = (\boldsymbol{I_0} + \ln(2)/\boldsymbol{\tau})(1-\exp(\boldsymbol{u}\, p_E)) \tag{7}$$

where $\boldsymbol{g}$ is the *domestication* rate constant measured in reciprocal years (y^{-1}), $p_E = ecosphere/E_0$, $\boldsymbol{\tau}$ is the initial doubling time of the population in years, and $\boldsymbol{I_0}$ is the initial value of *ignorance*. The negative-valued variable $\boldsymbol{u}$ models the aggressiveness of human growth as *ecosphere* approaches zero, discussed below. A large negative $\boldsymbol{u}$ means aggressive domestication. Flow of *humansphere* back to *ecosphere* is called *rewilding* (see Eq 4). Note that *ignorance,* a number that reflects the mortality rate, initially declines exponentially as *knowledge* takes its place.

Expressing human population using the total ecological footprint of humanity is functionally equivalent to the more traditional birth/death model. *Rewilding* is equivalent to death, since, upon death, a human's resources are returned to the *ecosphere*. Food that is not eaten counts as carbon sequestered, and waste that is not produced counts as waste assimilated. Therefore, death converts *humansphere* to *ecosphere*. By the same token, *domestication* is functionally equivalent to birth since it converts *ecosphere* to *humansphere* to support an increase in our numbers.

Knowledge (of technology) flows from *ignorance* (of technology), at the intrinsic rate called *learning* = $\boldsymbol{\kappa}$ **knowledge*. The optimal value $\boldsymbol{\kappa}$ =9.6e-3 y^{-1} was determined from the data. The amount of *learning* is the degree to which life expectancy has been increased by science and technology in a given year. Theoretically, maximum *knowledge* implies zero *ignorance*, which unrealistically implies zero death. However, simulations never come close to this value. On the other hand, zero *knowledge* implies a mortality rate of equal to the base value, estimated as $\boldsymbol{I_0}$=0.11 y^{-1}. Unfortunately $\boldsymbol{I_0}$ could not be precisely fit because it affects the simulations only after the present time. This estimate is derived from the estimated current overshoot of the global biocapacity[5].

Knowledge can and does flow back to *ignorance* in the sense that technology becomes lost or obsolescent. Obsolescence happens when new forms of mortality emerge from advancing technology, such as cancer arising from chemical synthesis of pesticides, draught arising from the efficient depletion of aquifers, and disease transmission arising from the increased ease of long-distance travel. In these cases, innovations that initially increased lifespan later uncovered a cryptic ignorance. To model cryptic ignorance, *knowledge* flows back to *ignorance* with value *obsolescence = r * knowledge,* where

$$r = exp(\boldsymbol{\nu}\, p_E) \tag{8}$$

where $p_E = ecosphere/E_0$. *Obsolescence* is increased by environmental degradation. The negative-valued variable $\boldsymbol{\nu}$ was fit to late 20th century population data. A larger negative value for $\boldsymbol{\nu}$ leads to less *obsolescence* and steeper population growth.

To achieve the observed population growth in the 20th century, *knowledge* was applied to both population growth and the growth of the carrying capacity (*CC*). In preliminary studies, applying knowledge to only the mortality rate (*rewilding*) or to only the *CC* did not reproduce the observed steepness of 20th century population growth. *CC* is defined as the number of people that can be supported on one global hectare (gha) of *humansphere*. The global biocapacity is treated as an unknown constant value $\boldsymbol{E_0}$, an amount of the world's biocapacity which is split between *ecosphere* and *humansphere*[12]. Both *ecosphere* and *humansphere* contribute resources needed for human life, therefore $CC = cc_E + cc_H$, but only the carrying capacity contributed by the *humansphere* (cc_H) is augmented by *knowledge*, as shown here.

$$cc_H = \boldsymbol{c}\ (1 - exp(\boldsymbol{d} * knowledge))\ cc_E \tag{9}$$

In this view, the carrying capacity contributed by the *humansphere* is wholly dependent on the *ecosphere* carrying capacity (cc_E, Eq 10 below), because food production approaches zero as ecosystem services approach zero. The loss of ecosystem services that maintain climate stability would leave us with unpredictable temperature, precipitation, and storms. Season-to-season instability and unpredictability limit the efficiency of agriculture. Thus environmental degradation (loss of *ecosphere*) leads to a decreased carrying capacity and therefore a loss of human population. Consider for example that the *ecosphere* includes fresh water aquifers and fossil fuels, factors in food production which are not likely to be replaced by any amount of human invention. Human food production is also limited by the total area of agricultural lands (*humansphere*) times the maximum carrying capacity of those lands under intense cultivation (***c***)[5]. The term 1-exp(***d*** * *knowledge*) expresses the rise in food production efficiency per gha as *knowledge* increases[4]. Following the "law of diminishing returns"[30], food production rises more slowly with each additional unit of knowledge. The optimal value for the degree of diminishing returns was $\boldsymbol{d} = -110$.

Although carrying capacity is wholly dependent on ecosystem services, those services are related to the biocapacity of the *ecosphere* in a way that cannot be easily assumed. We may hypothesize that damage to the ecosystem is not felt until a certain threshold in degradation is reached. Thereafter the damage to ecosystem services may be rapid and go asymptotically to zero, as described allegorically in Ehrlich's "The Population Explosion"[33] with the "rivet popper" story. Indeed, in many ways, the global ecosystem, like the airplane wing in the story, can take serious damage before it suddenly fails in flight. On the other hand, the environment may be more robust than we know and the *ecosphere* may pass a percentage of its ecosystem services to the *humansphere*, never really collapsing to zero. An asymmetric sigmoid function (Eq 10) is used to express a continuum of unknown non-linear relationships between *ecosphere* and ecosystem services (cc_E). The asymmetric sigmoid curve allows us to optimize a single parameter ($\boldsymbol{a}$) to express the degree of robustness or fragility of the ecosystem. If $\boldsymbol{a}$ is low**,** the environment responds robustly to depletion of *ecosphere* by retaining ecosystem services within the *humansphere*, whereas if $\boldsymbol{a}$ is high then the ecosystem is fragile and critical services are lost suddenly, as described in Ehrlich's story. This function allows to explore ecosystem fragility by fitting proposed non-linear ecosystem behavior to population.

$$cc_E = \boldsymbol{b}\, p_E^{(0.5/(1+p_E-2\boldsymbol{a}))} \qquad (10)$$

where p_E = *ecosphere*/$\boldsymbol{E_0}$. Figure 5 shows the shape of the cc_E function and how it changes with $\boldsymbol{a}$.

Figure 5. Plot of Eq 10, the *ecosphere* component of the carrying capacity for humans. Increasing values of $\boldsymbol{a}$ move the curve to the right, meaning ecosystem services are more fragile with respect to the *ecosphere*.

Variable $\boldsymbol{a}$ was initially set to a value that places the global ecosystem today at about one-half *humansphere* and one-half *ecosphere* in 2005, the year of "peak oil"[34], albeit that peak date should perhaps be pushed forward by new discoveries of natural gas and new mining technologies. Fossil fuel is a dominant resource in raising the carrying capacity in the 20th century and its numbers are well studied and readily available, however it should be recognized that other limiting natural resources and ecosystem services may "run out" before fossil fuel does. Therefore, the variable $\boldsymbol{a}$ was set, not by *a priori*, but by fitting to late 20th century population data as described below. This variable, along with $\boldsymbol{E_0}$, is one of the most critical to the accurate fitting of late 20th century, post-hyperexponential growth. The optimal value of $\boldsymbol{a}$ was found to be in a range from 0.27 to 0.48 depending other variables. Figure 3a,b,c shows the shape of the range of optimality in dimensions $\boldsymbol{a}$, $\boldsymbol{E_0}$, and $\boldsymbol{v}$.

It is not clear how all these values in (7) to (10) interact with (6). The model is poorly set out.

The optimized model (Table 1) was found to closely reproduce a 65% increase in the total caloric output of agriculture observed during the Green Revolution from 1960-2010[31,32]. In the model, the carrying capacity cc_H goes from 0.80 gha per capita in 1960, to 1.33 gha per capita in 2010, a 66% increase. The model also roughly reproduces the total human ecological footprint increase of 225% from 0.75 Earths in 1960 to 1.7 Earths in 2010[50]. In the model, *humansphere* grows 187% over the same period.

Model Fitting to Population Data

Variable setting were determined by least-squares fitting using the program *Hyperfit*. All variables in ***bold italics*** in Table 1 were fit to data. The most accurate and reliable data relevant to human population is the population itself. Global population numbers have been estimated for as far back as 10,000 years ago, and recent numbers are likely to be correct within 3%[19]. Exploratory curve fitting was used to test functional forms using Microsoft Excel, using the Solver plug-in[35]. Other global numbers, such as the ecological footprint, world forest cover, atmospheric carbon, global economic output, etc. were used in supporting roles only.

Several different methods were used for least-squares fitting. Initially, the parameters were fit heirarchically. The variables were fit to population data starting with the distant past and progressing forward to the present. Eight variables defining exponential and early hyper-exponential growth ($\boldsymbol{I_0, \tau_0, H_0, K_0, \kappa, b, c, d}$) were fit to population data from years 1000 to 1970 using Eq. 1. Non-linear least-squares (Solver[35]) was used to find the parametric optimum but failed to converge due to strong covariance. Finally, a range of least-squares minima was identified by exhaustive sampling of variables within the ranges defined by the heirachical fit method, using *Hyperfit*.

The model is over-specified
There are too many parameters.
Hence the problems with
parameter fitting.

Hyperfit is a program in Fortran90 that runs the World4 model for any number of ranges of variables, either randomly or exhaustively. In exhaustive sampling mode, *Hyperfit* reads an input file containing the ranges to be sampled and parameters for the range of years to be fit and the sampling density. The output is a matrix of mean square residuals summed over the range of years sampled. Up to 4 variables can be sampled exhaustively. The program outputs to the plotting program *gnuplot*[51]. In random sampling mode, *Hyperfit* samples all variables within ranges defined in the input file using a flat probability distribution, then runs a simulation and saves the parameters if the residual is below a cutoff (*e.g.* 0.5e8 for the years 1970-2010). The set of best fit parameters were passed back into *Hyperfit* to generate trajectories for plotting, using Microsoft Excel.

The growth model shown in Eq. 6 fits populations from 1000 to 1960 but does not fit populations from the most recent 50 years as shown in Figure 3d. In the last 50 years, population growth has slowed, going from hyper-exponential to sub-exponential to linear. The slowing of growth is rationalized as a carrying capacity effect, whose dependency on the environment and technology is modeled by Eqs 9 and 10. These equation contain four additional parameters which modify growth to match the data to within ±20 million or 0.3%. The four variables ($\boldsymbol{a, E_0, u, v}$) do not have a significant effect on populations before the year 1970. An additional four variables (***w, y, py, sy***) were defined for *what-if* scenarios only, and these were not fit to data and do not affect population prior to 2020.

This description is totally unclear.

Model availability

The interactive model World4 may be accessed on the InsightMaker web site[36]. By cloning the model, anyone can make changes. Complete parameters and equations for the model are included in the model itself and in Table 1. The program *Hyperfit* is freely available from the author upon request. *Hyperfit* requires installation of *gnuplot*.[51]

Acknowledgements

Several people have provided feedback on this manuscript in its current form or in one of its earlier forms, including Robert Wyman, George Richardson, and Lee Badger. Special thanks to the students who took the RPI computational biology course Human Population and tested several alternative models as part of their homework assignments.

References

1. Lutz, W., Sanderson, W., & Scherbov, S. (2001). The end of world population growth. Nature, 412(6846), 543-545.
2. Hopfenberg, R. (2003). Human carrying capacity is determined by food availability. Population and environment, 25(2), 109-117.
3. Varfolomeyev, S. D., & Gurevich, K. G. (2001). The hyperexponential growth of the human population on a macrohistorical scale. Journal of Theoretical Biology, 212(3), 367-372.
4. Kremer, M. (1993). Population growth and technological change: One million BC to 1990. The Quarterly Journal of Economics, 108(3), 681-716.
5. Lin, D., et al. (2018). Ecological footprint accounting for countries: updates and results of the national footprint accounts, 2012–2018. Resources, 7(3), 58.
6. United Nations, Model-Based Estimates and Projections of Family Planning Indicators: 2019 Revision (Population Division, Department of Economic and Social Affairs, United Nations, New York, 2014).
7. Cohen, J. E. (1995). Population growth and earth's human carrying capacity. Science, 269(5222), 341-346.
8. Fanzo, J., Davis, C., McLaren, R., & Choufani, J. (2018). The effect of climate change across food systems: Implications for nutrition outcomes. Global food security, 18, 12-19.

9. Bongaarts, J. (2009). Human population growth and the demographic transition. Philosophical Transactions of the Royal Society B: Biological Sciences, 364(1532), 2985-2990.

10. Von Foerster, H., Mora, P. M., & Amiot, L. W. (1960). Doomsday: Friday, 13 november, ad 2026. Science, 132(3436), 1291-1295.

11 Wackernagel, M., et al. (1999). National natural capital accounting with the ecological footprint concept. Ecological economics, 29(3), 375-390.

12. Wackernagel, M., & Rees, W. (1998). Our ecological footprint: reducing human impact on the earth (Vol. 9). New society publishers.

13. Forrester, J. W. (1971). World dynamics. Wright-Allen Press.

14. Meadows, D. H., Meadows, D. L., Randers, J., & Behrens, W. W. (1972). The limits to growth. New York, 102(1972), 27.

15. Meadows, D., Randers, J., & Meadows, D. (2004). Limits to growth: The 30-year update. Chelsea Green Publishing.

16. Linneman, H., Hoogh, J. D., Keyzer, M. A., & Van Heemst, H. D. (1979). MOIRA: Model of International Relations in Agriculture. Report of the project group" Food for a doubling world population". North Holland Publishing Comp.

17 . Sverdrup, H. U., & Ragnarsdottir, K. V. (2011). Challenging the planetary boundaries II: Assessing the sustainable global population and phosphate supply, using a systems dynamics assessment model. Applied Geochemistry, 26, S307-S310.

18. Bardi, U. (2013). Mind sized world models. Sustainability, 5(3), 896-911.

19. Keilman, N. (2001). Data quality and accuracy of United Nations population projections, 1950-95. Population Studies, 55(2), 149-164.

20. Maystadt, J. F., & Ecker, O. (2014). Extreme weather and civil war: does drought fuel conflict in Somalia through livestock price shocks?. American Journal of Agricultural Economics, 96(4), 1157-1182.

21. Messer, E. (2009). Rising food prices, social mobilizations, and violence: conceptual issues in understanding and responding to the connections linking hunger and conflict. Napa Bulletin, 32(1), 12-22.

22. Epule, T. E., Ford, J. D., Lwasa, S., & Lepage, L. (2017). Climate change adaptation in the Sahel. Environmental Science & Policy, 75, 121-137.

23 Fisseha, M. (2019). Baking desert, cold reception? Climate change driven migration and international legal protection: the case study of the Sahel region.

24. Gustafson, D. (2020). Hunger to Violence: Explaining the Violent Escalation of Nonviolent Demonstrations. Journal of Conflict Resolution, 0022002719890669.

25. Bardi, U. (2011). The Seneca effect: why decline is faster than growth. (https://thesenecatrap.blogspot.com/2015/11/the-seneca-effect-why-decline-is-faster.html (Accessed 8/23/20).

26. Weiwei A. (director, producer) and Yap, C-C. (producer), Deckert, H. (producer) & Cohen, A. (producer) (2017) Human Flow[DVD] Germany: AC Films, Participant Media.

27. Jackson, T. (2009) Prosperity Without Growth: Economics for a Finite Planet. New York, NY: Earthscan, Taylor & Francis Routledge.

28. Ehrlich, P.R. and J.P. Holdren (1971) 'Impact of Population Growth: Complacency concerning this component of man's predicament is unjustified and counterproductive'. Science 171(3977): 1212-1217.

29. FAO, IFAD, UNICEF, WFP and WHO. 2020. The State of Food Security and Nutrition in the World 2020. Transforming food systems for affordable healthy diets. Rome, FAO. https://doi.org/10.4060/ca9692en

30. https://en.wikipedia.org/wiki/Diminishing_returns (Accessed 8/27/20)

31. Pingali, P. L. (2012). Green revolution: impacts, limits, and the path ahead. Proceedings of the National Academy of Sciences, 109(31), 12302-12308.

32. Awika, J. M. (2011). Major cereal grains production and use around the world. In Advances in cereal science: implications to food processing and health promotion (pp. 1-13). American Chemical Society.

33. Ehrlich P. and A. Ehrlich. 1990. The Population Explosion. New York: Simon and Schuster.

34. Bardi, U. (2009). Peak oil: The four stages of a new idea. Energy, 34(3), 323-326.

35. Harris, D. C. (1998). Nonlinear least-squares curve fitting with Microsoft Excel Solver. Journal of chemical education, 75(1), 119.

36. https://insightmaker.com/insight/157730/World4-5 (Accessed 8/27/20)

37. Wilson, E. O. (2016). Half-earth: our planet's fight for life. WW Norton & Company.

38. Rooney-Varga, J. N., Kapmeier, F., Sterman, J. D., Jones, A. P., Putko, M., & Rath, K. (2020). The climate action simulation. Simulation & Gaming, 51(2), 114-140.

39. United Nations, Department of Economic and Social Affairs, Population Division (2019). World Population Prospects 2019: Highlights (ST/ESA/SER.A/423). https://population.un.org/wpp/Publications/Files/WPP2019_Highlights.pdf (accessed 8/27/20)

40. Coccia, M. (2014). Driving forces of technological change: the relation between population growth and technological innovation: analysis of the optimal interaction across countries. Technological Forecasting and Social Change, 82, 52-65.

41. Strulik, H. (2005). The role of human capital and population growth in R&D-based models of economic growth. Review of international Economics, 13(1), 129-145.

42. Madsen, J. B., Ang, J. B., & Banerjee, R. (2010). Four centuries of British economic growth: the roles of technology and population. Journal of Economic Growth, 15(4), 263-290.

43. Kurzweil, R. (2014). The singularity is near. In *Ethics and emerging technologies* (pp. 393-406). Palgrave Macmillan, London.

44. Coole, D. (2013). Too many bodies? The return and disavowal of the population question. *Environmental Politics*, *22*(2), 195-215

45. Cohen, J. E. (1996). *How many people can the earth support?*. WW Norton & Company.

46. Korotayev, A. V. (2020). The Twenty-First-Century Singularity in the Big History Perspective—A Re-analysis. In *The 21st Century Singularity and Global Futures* (pp. 19-75). Springer, Cham.
47. Ehrlich, P. R. (2014). Human impact: the ethics of I= PAT. *Ethics in Science and Environmental Politics*, *14*(1), 11-18.
48. Cliggett, L. (2001). Carrying capacity's new guise: folk models for public debate and longitudinal study of environmental change. *Africa Today*, 3-19.
49. https://www.discovery.org/c/human-exceptionalism/ (accessed 22 Jan 2021)
50. Lin, D., Hanscom, L., Murthy, A., Galli, A., Evans, M., Neill, E., ... & Wackernagel, M. (2018). Ecological footprint accounting for countries: updates and results of the national footprint accounts, 2012–2018. *Resources*, *7*(3), 58.
51. Williams T, Kelley C, Bröker HB, Campbell J, Cunningham R, Denholm D, Elber G, Fearick R, Grammes C, Hart L, Hecking L. gnuplot 4.6. An Interactive Plotting Program. 2012.

Figure 1

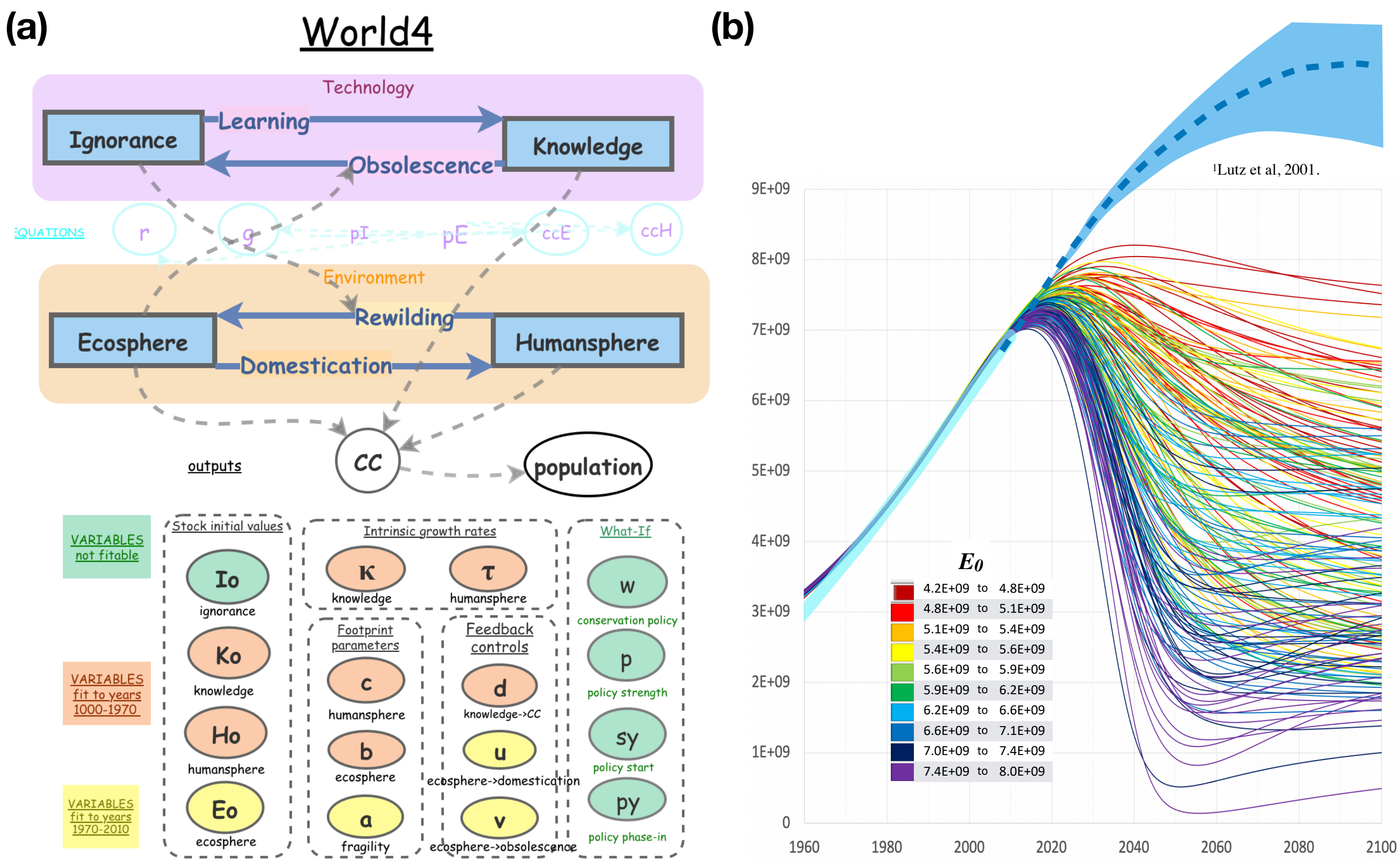


Figure 1

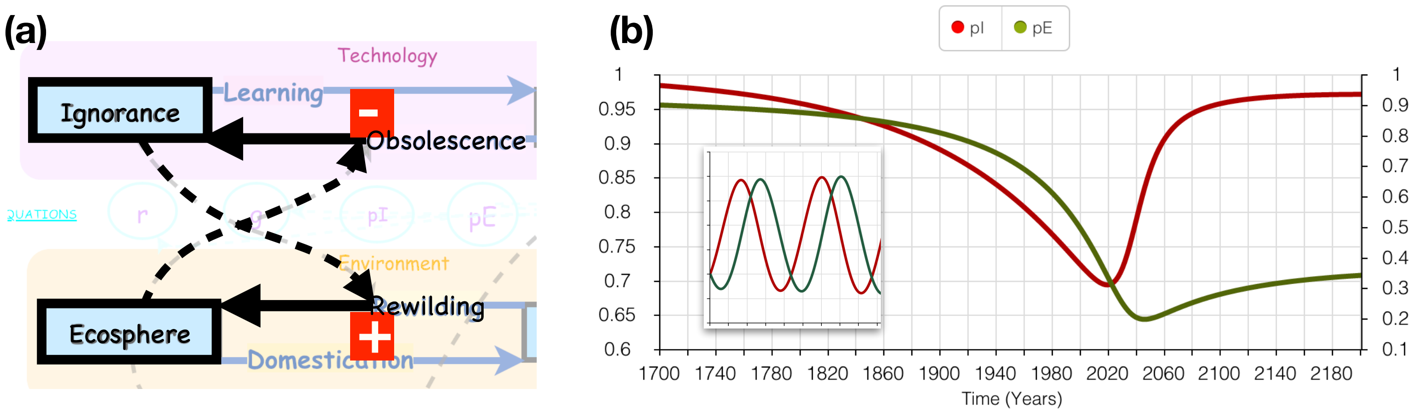

(a)
Technology
Ignorance
Learning
-
Obsolescence
EQUATIONS
r
g
pI
pE
Environment
Rewilding
+
Ecosphere
Domestication
(b)
pI
pE
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
1700 1740 1780 1820 1860 1900 1940 1980 2020 2060 2100 2140 2180
Time (Years)

Figure 2

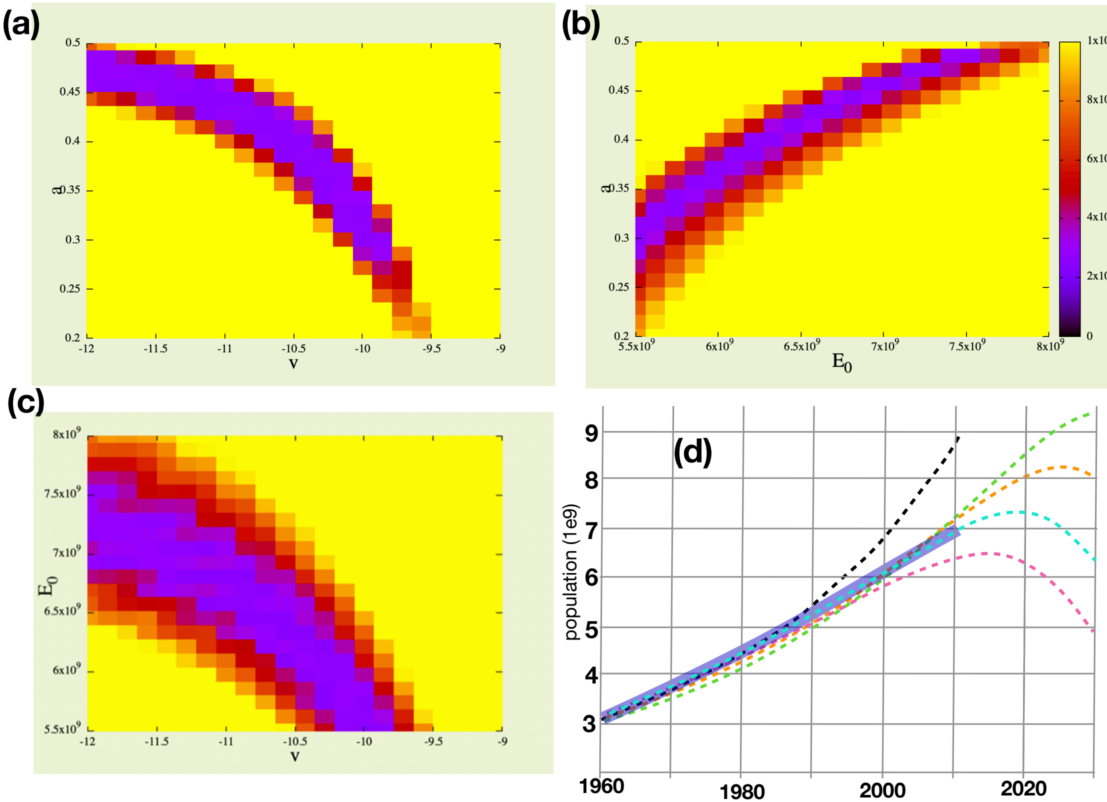

(a)
0.5
0.45
0.4
a
0.35
0.3
0.25
0.2
-12
-11.5
-11
-10.5
-10
-9.5
-9
v
(b)
0.5
0.45
0.4
a
0.35
0.3
0.25
0.2
5.5x10^9
6x10^9
6.5x10^9
7x10^9
7.5x10^9
8x10^9
E_0
1x10
8x10
6x10
4x10
2x10
0
(c)
8x10^9
7.5x10^9
7x10^9
E_0
6.5x10^9
6x10^9
5.5x10^9
-12
-11.5
-11
-10.5
-10
-9.5
-9
v
(d)
9
8
7
6
5
4
3
population (1e9)
1960
1980
2000
2020

Figure 3

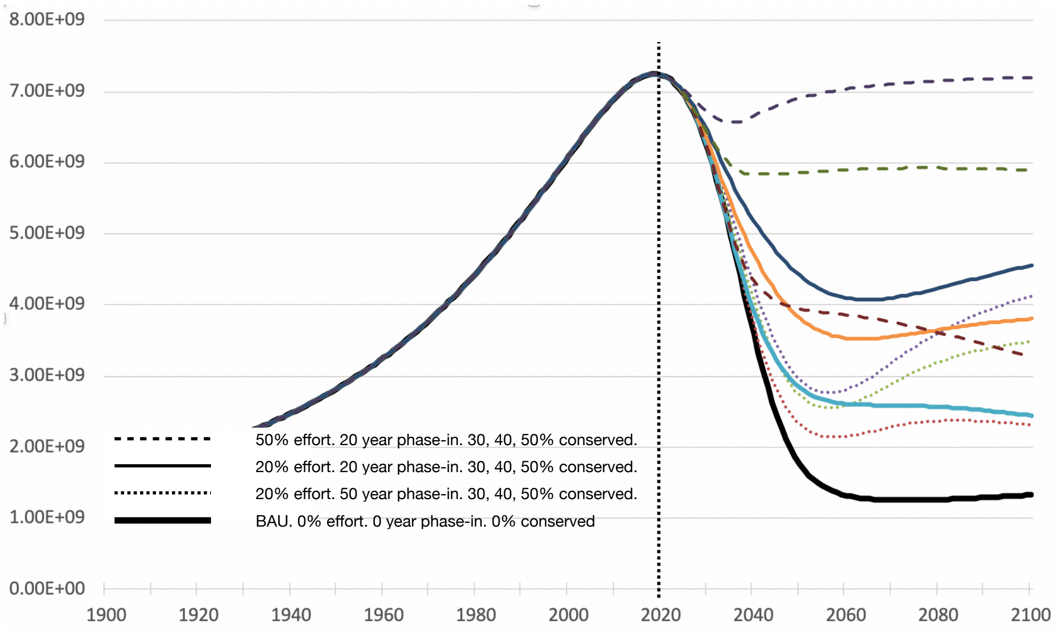
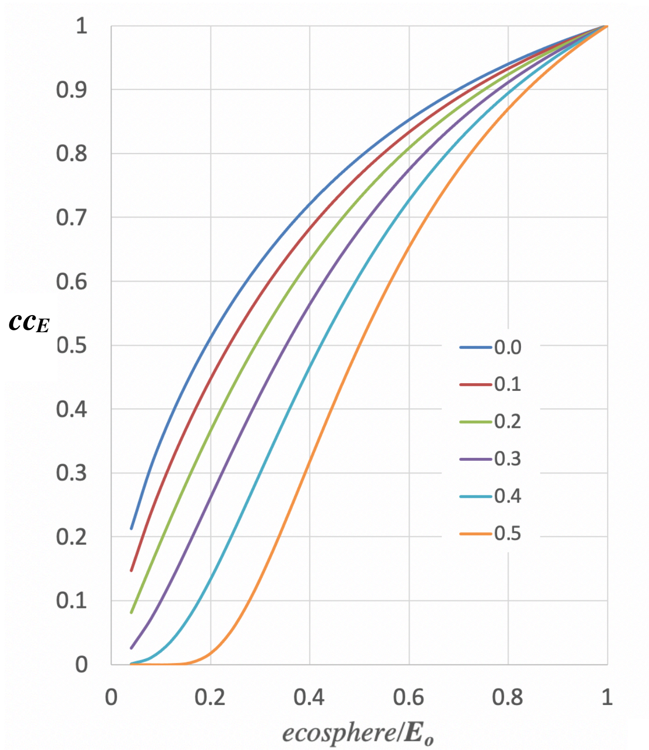

8.00E+09
7.00E+09
6.00E+09
5.00E+09
4.00E+09
3.00E+09
2.00E+09
1.00E+09
0.00E+00
1900
1920
1940
1960
1980
2000
2020
2040
2060
2080
2100
50% effort. 20 year phase-in. 30, 40, 50% conserved.
20% effort. 20 year phase-in. 30, 40, 50% conserved.
20% effort. 50 year phase-in. 30, 40, 50% conserved.
BAU. 0% effort. 0 year phase-in. 0% conserved

Figure 4



1
0.9
0.8
0.7
0.6
cc_E
0.5
0.4
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
1
$ecosphere/\mathbf{E}_o$
0.0
0.1
0.2
0.3
0.4
0.5

Figure 5